Analytical study on coordinative optimization of convection in tubes with variable heat flux

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Received July 5, 2004

Abstract The laminar heat transfer in the thermal entrance region in round tubes, which has a variable surface heat flux boundary condition, is analytically studied. The results show that the heat transfer coefficient is closely related to the wall temperature gradient along the tube axis. The greater the gradient, the higher the heat transfer rate. Furthermore, the coordination of the velocity and the temperature gradient fields is also analysed under different surface heat fluxes. The validity of the field coordination principle is verified by checking the correlation of heat transfer coefficient and the coordination degree. The results also demonstrate that optimizing the thermal boundary condition is a way to enhance heat transfer.

Keywords: convective heat transfer, entrance region, field coordination.

DOI: 10.1360/03ye0502

It was certified that the heat transfer between a fluid and a solid wall is closely related to the coordination of the velocity and the temperature gradient/heat flux fields[1,2]. When the two fields in a system are fully coordinated, the heat transfer is optimized. Then the highest heat transfer rate is achieved with the least work consumption of fluid flow. Corresponding to the difference of coordination degree, Nusselt number, which indicates the strength of heat transfer, can change between $Nu = Re \cdot Pr$ and $Nu = 0$[3]. This theory is referred to as field coordination principle (FCP).

The significance of FCP lies in that it tells us what is the maximum of the heat transfer rate for a configuration under a certain condition (velocity, temperature, physical properties, etc.). Therefore it is an important theory not only for heat transfer science itself, but also for transfer enhancement technology. Guo et al. studied the mechanism of convection in a boundary layer and checked the coordination of the fields, and then they proposed some new ways to enhance convective heat transfer: (1) use of gradully contracting ducts; (2) variation of thermal boundary conditions; and (3) use of specially designed inserts. The present work focuses on the way of varying thermal boundary condition. The main purpose is to find some special surface heat flux distributions that improve the heat transfer. The discussed object in the following is restricted to the laminar heat transfer in round tubes, in which the velocity field is fully developed.

1 Wall temperature responses to exponential surface heat flux distributions

The coordination degree of the velocity and the temperature gradient fields depends mainly on their included angle. For 2-D convective heat transfer, the vector dot product of the two fields is

$$\vec{V}\cdot\nabla T = u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}. \tag{1}$$

For tubal flow, x and y are the axial and the radial coordinator, respectively; u and v are their corresponding velocity component. For a smooth duct, there is no velocity component in y direction in fully developed region. Therefore the field coordination degree is equal to $u.\ \partial T/\partial x$, i.e. depends on the axial temperature gradient. This gradient on wall is zero under uniform wall temperature (UT) condition, and a constant under uniform wall heat flux (UH) condition. Now the problem is: find some special surface heat flux distributions, which give greater temperature gradient in axial than UH, and determine the corresponding heat transfer coefficient.

As seen in the following, exponential distribution of surface heat flux is able to generate a series of temperature responses of different gradients, which includes UT and UH conditions. In the following, this kind of distribution will be taken as the discussed object. Assuming the velocity field is fully developed, the velocity profile takes a parabolic shape. The flowing fluid in the tube is being heated with a heat flux distribution as

$$q_w(\xi) = q_0 \mathrm{e}^{\frac{1}{2}m\xi}, \tag{2}$$

where $\xi = (x/r_0)/(Re \cdot Pr)$, denoting the dimensionless axial distance, in which r_0 is the radius of the tube, and m is an arbitrary constant. The temperature response to the heat flux distribution is determined as[4]

$$T_w(\xi) - T_{in} = \frac{r_0}{k}\int_0^{\xi}\left(4 + \sum_{n=1}^{\infty}\frac{\exp(-\gamma_n^2(\xi-\xi_i))}{\gamma_n^2 A_n}\right)q_w(\xi_i)\mathrm{d}\xi_i. \tag{3}$$

In eq. (3) T_{in} is the inlet temperature of the fluid, and k is the conductivity. γ_n and A_n are the engine value and its coefficient in the solution of energy equation under UH condition (see table 1)[5].

Table 1 Engine value and its coefficient

n	1	2	3	4	5	6	7	8	9
γ_n^2	25.68	83.86	174.2	296.5	450.9	637.4	855.9	1111.1	1393.8
$A_n \times 10^3$	7.631	2.053	0.903	0.494	0.307	0.207	0.148	0.120	0.092

Substituting eq. (2) into eq. (3), we have

$$\theta_w(\xi) = \frac{T_w(\xi) - T_{in}}{\frac{q_0 r_0}{k}} = \int_0^{\xi} \left(4 + \sum_{n=1}^{\infty} \frac{\exp(-\gamma_n^2(\xi - \xi_i))}{\gamma_n^2 A_n} \right) \cdot \exp\left(\frac{m\xi_i}{2} \right) \mathrm{d}\xi_i. \tag{4}$$

Integrating the right part of the above equation, the relation between θ and ξ is

$$\theta_w(\xi) = \sum_{n=1}^{\infty} \frac{-2\gamma_n^2}{2\gamma_n^2 + m} \cdot \frac{\exp\left(-\gamma_n^2 \xi\right)}{\gamma_n^4 A_n} + \sum_{n=1}^{\infty} \frac{-m}{2\gamma_n^2 + m} \cdot \frac{\exp\left(\frac{1}{2} m\xi\right)}{\gamma_n^4 A_n}$$
$$+ \frac{8}{m}\left[\exp\left(\frac{1}{2} m\xi \right) - 1 \right] + \frac{11}{24} \exp\left(\frac{1}{2} m\xi \right). \tag{5}$$

After checking eq. (5), it is revealed that the case of $m = -2\gamma_0^2 = -14.63$, in which γ_0 is the first engine value in the solution of energy equation under UT condition, tends to be a uniform temperature distribution when ξ is large enough ($\xi > 0.1$), while the case of $m = 0$ represents the temperature response of UH.

The variation of the wall temperature described in eq. (5) is plotted in fig. 1. It can be seen that for $m < 0$, θ_w reaches a constant value as ξ increases, while for $m \geqslant 0$ θ_w reaches infinite eventually.

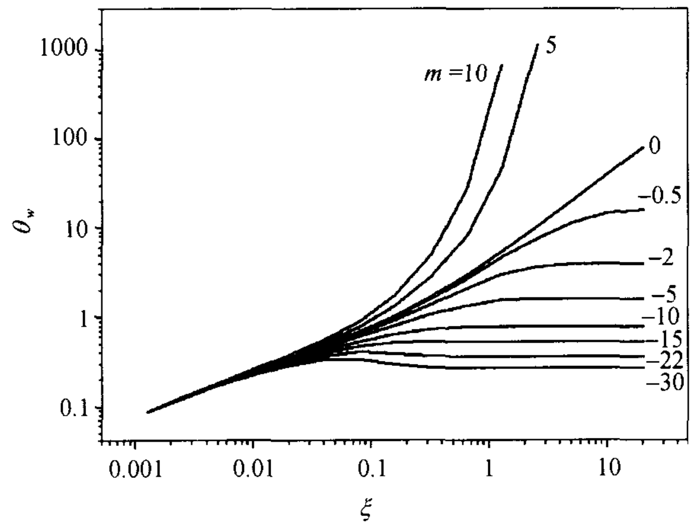

Fig. 1. Variation of the wall temperature in the axial direction.

2 Distribution of the Nusselt number

2.1 Nusselt number of the entrance region

For the convective heat transfer in ducts, the relation between the local heat flux and the difference of temperature is

$$q_w(\xi) = h(\xi)[T_w(\xi) - T_b(\xi)]. \tag{6}$$

By the principle of energy balance, the local bulk temperature of fluid $T_b(\xi)$ can be determined as

$$T_b(\xi) = T_{in} + \frac{4r_0}{k}\int_0^{\xi} q_w(\xi)d\xi.$$

For the present problem, substituting eq. (2) into the above equation, we obtain

$$T_b(\xi) = T_{in} + \frac{8r_0 q_0}{km}\left(\exp\left(\frac{1}{2}m\xi\right) - 1\right). \tag{7}$$

Substitute eq. (7) into eq. (6), and combine eq. (2) and eq. (5). Then the local Nusselt number is derived as

$$Nu(\xi) = \frac{2r_0 \cdot h(\xi)}{k} = \frac{2}{B}, \tag{8}$$

where

$$B = \sum_{n=1}^{\infty} \frac{-2\gamma_n^2}{2\gamma_n^2 + m} \cdot \frac{\exp(-(\gamma_n^2 + m/2)\xi)}{\gamma_n^4 A_n} + \sum_{n=1}^{\infty} \frac{-m}{2\gamma_n^2 + m} \cdot \frac{1}{\gamma_n^4 A_n} + \frac{11}{24}.$$

Fig. 2 shows the changes of $Nu(\xi)$ with ξ at different m values. It is readily seen the difference of the entrance and the fully developed region. For a fixed m, $Nu(\xi)$ decreases linearly at first. After a transition region, it reaches a constant and keeps constant thereafter. An m value corresponds to a constant of $Nu(\xi)$. The greater the m, the higher the constant. Another point worthy to note is: with different m values, the entrance lengths are different, too (see fig. 3). Here the entrance length is defined as the distance from the inlet where the $Nu(\xi)$ is equal to 1.03 times that of fully developed region. As m in-

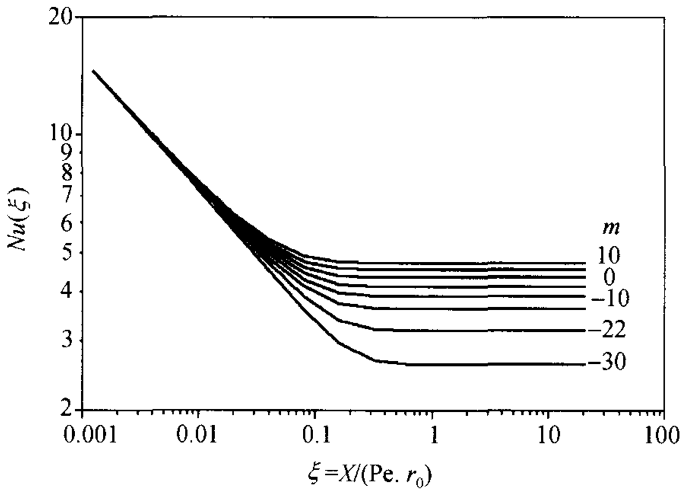

Fig. 2. Local Nusselt numbers as q_w changes as $q_w(\xi) = q_0 e^{m\xi/2}$.

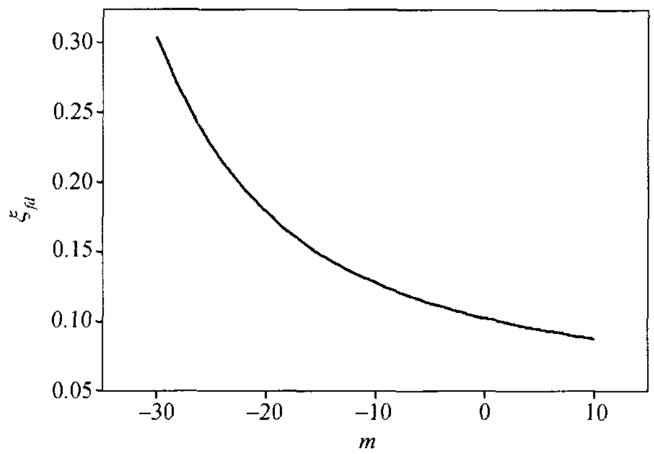

Fig. 3. Entrance length at different exponent value *m*.

creases gradually from –30 to 10, the dimensionless entrance length ξ_{fd} changes from 0.3 to 0.08 correspondingly. This indicates that the distribution of heat flux in the flow direction has strong effect on the length of entrance.

2.2 Nusselt number of the fully developed region

From eq. (8), when $m/2 + \gamma_1^2 > 0$, ($m > -2\gamma_1^2 = -51.36$), and if $\xi \to \infty$, we get

$$Nu_{fd} = \frac{2}{\dfrac{11}{24} + \displaystyle\sum_{n=1}^{\infty} \frac{-m}{2\gamma_n^2 + m} \cdot \frac{1}{\gamma_n^4 A_n}}. \tag{9}$$

On the other hand, when $m < -2\gamma_1^2 = -51.36$, $Nu(\xi)$ approaches zero as $\xi \to \infty$. It will be seen in the following that when $m < 0$ the temperature difference tends to be zero as $\xi \to \infty$. This indicates that $m = -51.36$ is a limit value for convective heat transfer with a surface heat flux distribution as eq. (2). For *m* less than this value there is no heat transfer in the fully developed region, meaning the flow in the duct is adiabatic.

The change of Nusselt number in the fully developed region for different *m* is presented in fig. 4. It is clear that the *Nu* of the fully developed region rises as *m* increases. This verifies the inference in ref. [1] that heat transfer can be enhanced when heat flux is escalating in the flow direction. It should be noted here that increase of the wall heat flux is only a part of its continuous change, which includes decreasing, uniform, and increasing of the heat flux axially. UH and UT conditions are simply special cases in the continuous change of wall heat flux. At the same time with the increasing heat flux, wall temperature climbs higher (see fig. 1). Excessively high temperature cannot be tolerated in practical engineering. Fig. 4 also shows that *Nu* increases with *m* nonlinearly. The increasing rate slows down as *m* is greater and greater. This is in contrast to the drastic climbing of wall temperature for $m > 0$ in fig. 1.

Shah and London[6] discussed a problem similar to the present work, and gave a correlation of *Nu* in the range of $-51.36 < m < 100$. Their correlation is

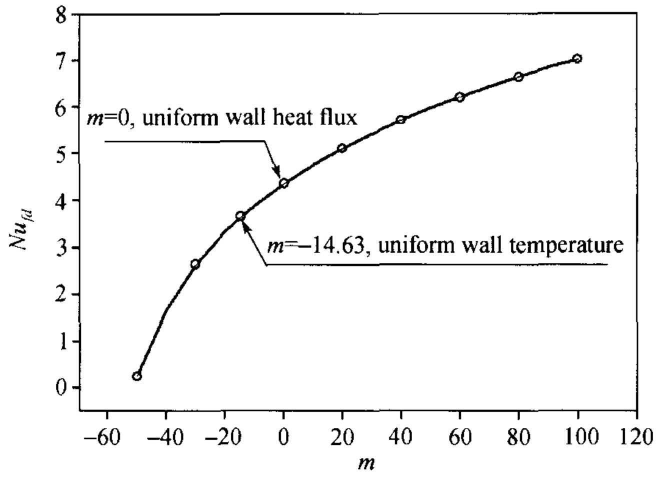

Fig. 4. *Nu* number of the fully developed region changes with *m*.

$$\begin{aligned} Nu = &\ 4.3573 + 0.0424m - 2.8368\times10^{-4}m^2 + 3.6250\times10^{-6}m^3 \\ &- 7.6497\times10^{-8}m^4 + 9.1222\times10^{-10}m^5 - 3.8446\times10^{-12}m^6. \end{aligned} \tag{10}$$

This correlation may be convenient for practical use.

3 Coordinative degree of fully developed region

According to eq. (1), coordinative degree (CD) is in connection with the vector dot product of velocity and temperature gradient. In consideration of the unchangeable *u* velocity profile in the fully developed region, and zero radial velocity correspondingly, the CD is therefore determined by the axial temperature gradient. On the other hand, the axial temperature gradient of fluid is mainly controlled by the variation of the axial wall temperature. Thus, we can take the variation of the axial wall temperature to roughly represent the CD in the vicinity of the solid wall.

By eq. (5), the axial gradient of wall temperature is derived as

$$\begin{aligned} \frac{\mathrm{d}\theta_w}{\mathrm{d}\xi} = &\sum_{n=1}^{\infty}\frac{2\gamma_n^4}{2\gamma_n^2+m}\cdot\frac{\exp(-\gamma_n^2\xi)}{\gamma_n^4 A_n} + \sum_{n=1}^{\infty}\frac{-m^2/2}{2\gamma_n^2+m}\cdot\frac{\exp\left(\frac{1}{2}m\xi\right)}{\gamma_n^4 A_n} \\ &+4\cdot\exp\left(\frac{1}{2}m\xi\right) + \frac{11}{48}m\cdot\exp\left(\frac{1}{2}m\xi\right). \end{aligned} \tag{11}$$

For different *m*, there occur three situations of $\dfrac{\mathrm{d}\theta_w}{\mathrm{d}\xi}$ as $\xi\to\infty$,

1) $-2{\gamma_1}^2 < m < 0,\ \dfrac{\mathrm{d}\theta_w}{\mathrm{d}\xi} = 0,$

2) $m = 0, \ \dfrac{d\theta_w}{d\xi} = 4,$

3) $m > 0, \ \dfrac{d\theta_w}{d\xi} \to \infty.$

These results are consistent qualitatively with those of analysis to the whole field in the tube. By aforementioned results, in addition to the limited *Nu* shown in fig. 4 and the heat flux described in eq. (2), the relation between the fluid bulk temperature T_b and the wall temperature T_w can be inferred as: As ξ increases, T_b is able to catch up with T_w when $m < 0$; T_b and T_w keep a fixed gap when $m = 0$; and, the difference of T_b and T_w is greater and greater when $m > 0$. This inference is verified when the difference of θ_w and θ_b is plotted in fig. 5. For $-2\gamma_1^2 < m < 0$ (fig. 5(a)), though the heat transfer coefficient is non-zero, the heat flux approaches zero because the temperature difference tends to be zero. On the other hand, the temperature difference approaches infinite for $m > 0$ (fig. 5(b)). This tells us that the increased heating load in the flow direction is absorbed by the

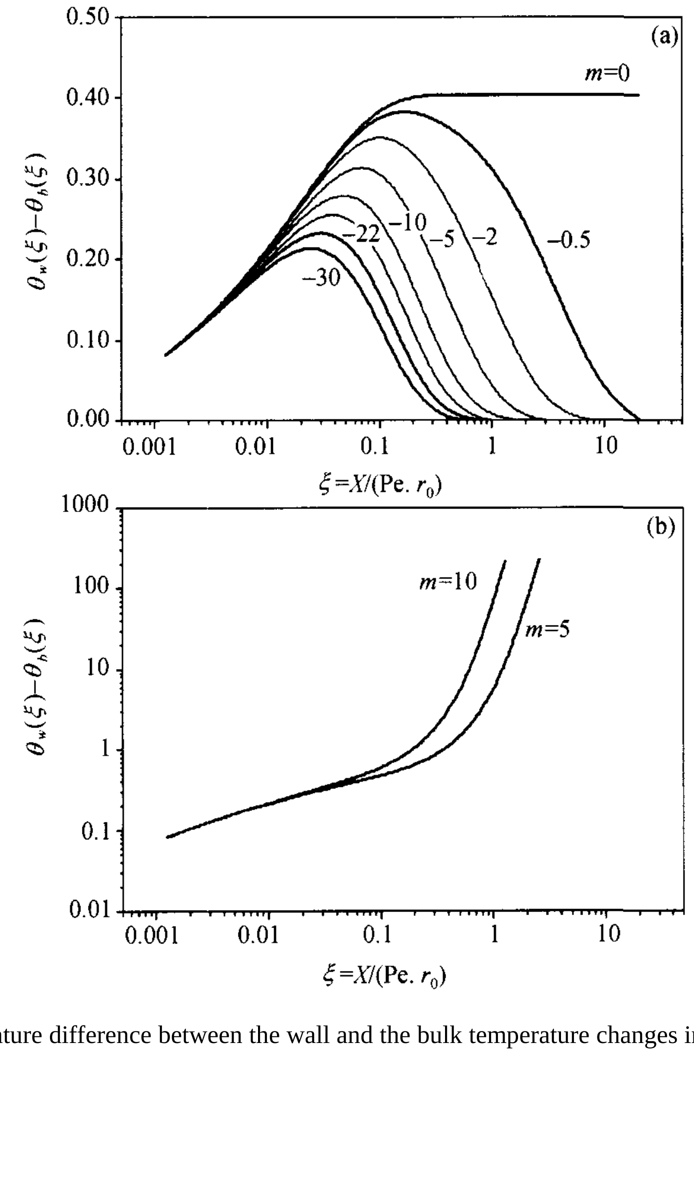

Fig. 5. Temperature difference between the wall and the bulk temperature changes in the flow direction.

fluid in two different mechanisms. One is active mechanism, in which the fluid regulates its temperature distribution to improve heat transfer coefficient. This is of benefit in terms of efficient transfer of thermal energy. The second mechanism is passive, which means the fluid enlarges its temperature difference with the solid wall to increase the heat transfer amount. This is undesirable for practical application. Heat transfer is a typical process of irreversibility, so that the loss of the useful part of energy is more as the temperature difference increases. In addition, increasing temperature difference can result in excessively high temperature of the tube wall.

Upon analyzing the cases of $m > 0$, though the heat transfer is enhanced partly by the active mechanism, the passive way plays a more important role in it. Therefore, this type of enhancement may not be easily applied in practical engineering. On the other hand, while decreasing the heat flux shows a relatively low heat transfer coefficient, it may be convenient for application because the resulted wall temperature is limited.

4 Conclusions

The laminar convective heat transfer in a round tube, to whose wall an exponential distribution of heat flux is applied, has been analytically solved. The result verifies the statement that increasing heat flux gradually in the flow direction can enhance the heat transfer. It is revealed that the heat transfer coefficient has a direct connection with the axial gradient of wall temperature. The greater the axial gradient is, the higher the heat transfer rate. Uniform wall temperature and uniform wall heat flux are simply special thermal condition cases in a series of heat flux distributions. Escalating heat flux gives a higher coefficient than uniform heat flux condition, though sharp growth of wall temperature will result. In comparison, decreasing the heat flux distribution is feasible to balance both the heat transfer rate and the wall temperature.

Acknowledgements The project is sponsored by the National Key Fundamental Research & Development Program (Grant No. G20000263), the Scientific Research Foundation for the Returned Overseas Chinese Scholars, State Education Ministry, and Beijing Municipal Commission of Education Program (Grant No. KM200410005020).

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